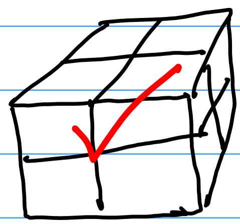
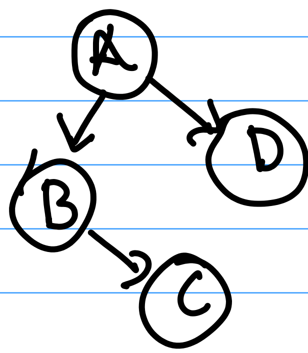
# CS181 - Inference in Bayesian Networks



Notes: HW5 due tomorrow!
- practical partnerships due too!
- Section Notes, Ed post on D-sep

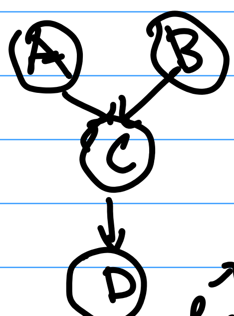
Last time - Bayesian Networks: encode structure (independence / conditional independence) of a joint distribution.



Key idea: D-sep

- $A \to C \to B$ $\quad B \leftarrow C \leftarrow A$
- $A \leftarrow C \to B$ — obs C blocks path
- $A \to C \leftarrow B$, $C \to D$ — not obs C, D preserves indep.

This time: inference in BN's.



- each node can take $\underline{K}$ values
- compute marginal prob:

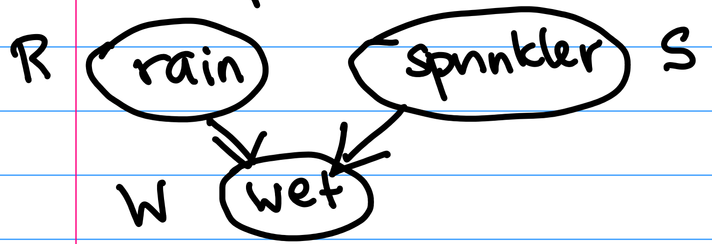
$$\underset{\text{for each } k}{P(D)} = \sum_{A,B,C} p(A,B,C,D) \quad \Big\} \; K^4 \text{ values}$$

$$= \sum_{A,B,C} p(A)\,p(B)\,p(C|A,B)\,p(D|C)$$

Goals: 1) how to compute queries like above?
2) how to compute these efficiently?

{Goal #1}

Simple example w/ queries:




• $p(S) = 1/2$ • $p(W|S,R)$:

• $p(R) = 1/4$

| S | R | $P(W\|..)$ |
|---|---|---|
| 1 | 1 | 99/100 |
| 1 | 0 | 9/10 |
| 0 | 1 | 9/10 |
| 0 | 0 | ∅ |

Q1) $$p(\underbrace{R=1}_{\text{query}} | \underbrace{W=1}_{\text{evidence}}) = \frac{P(R=1, W=1)}{P(R=1,W=1) + P(R=0,W=1)}$$

$$= \frac{P(R=1,W=1,S=1) + P(R=1,W=1,S=0)}{P(R=1,W=1,S=1) + P(R=1,W=1,S=0) + P(R=0,W=1,S=1) + P(R=0,W=1,S=0)}$$

$$= \frac{(1/4)(1/2)(99/100) + (1/4)(1/2)(9/10)}{(\frac{1}{4})(\frac{1}{2})(\frac{99}{100}) + (\frac{1}{4})(\frac{1}{2})(\frac{9}{10}) + (\frac{3}{4})(\frac{1}{2})(\frac{9}{10}) + (\frac{3}{4})(\frac{1}{2})(0)}$$

$\approx .4$

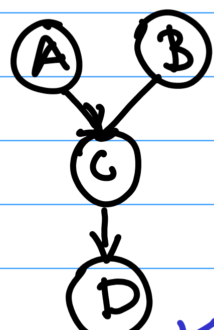
Q2) $$p(R=1 | W=1, S=1) = \frac{P(R=1,W=1,S=1)}{P(R=1,W=1,S=1) + P(R=0,W=1,S=1)}$$

$= 11/41 \approx .25$

"explaining away effect"

How can we compute these queries efficiently? (Goal #2)

Return:




$$\underline{\underline{p(D)}} = \sum_{A,B,C} \overbrace{p(A)p(B)p(C|A,B)\,p(D|C)}^{P(A,B,C,D)}$$

$$= \sum_C \underbrace{p(D|C)}_{K^2} \sum_B \underbrace{p(B)}_{K} \sum_A p(A) \underbrace{p(C|A,B)}_{K^3}$$

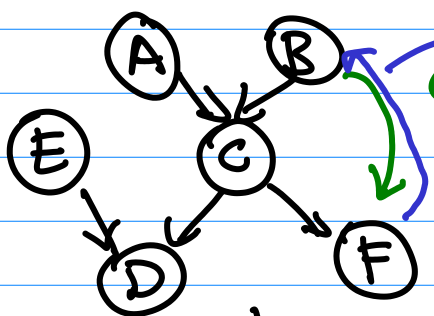
$K, g(D)$ $K, g(C)$ $K^2, g(C,B)$

Note: biggest factor was size $K^3$ — did <u>not</u> need to deal w/ $K^4$ tensor of joint probs!

Note: if we tried to $\sum_C$ first, wouldn't have helped:

$$= \sum_A p(A) \sum_B p(B) \sum_C \underbrace{p(D|C)\,p(C|A,B)}_{\text{size } K^4 \text{ !!}}$$

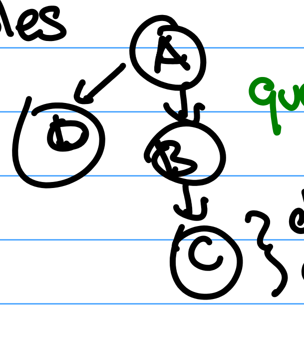
Overall, finding the optimal ordering is hard.
But, for some cases, there exists a strategy:
specifically: polytrees - no loops in the structure




polytree - no loops!

For polytree DAGs: to do inference:

① prune any vars that are not ancestors of the queried variable or evidence variables



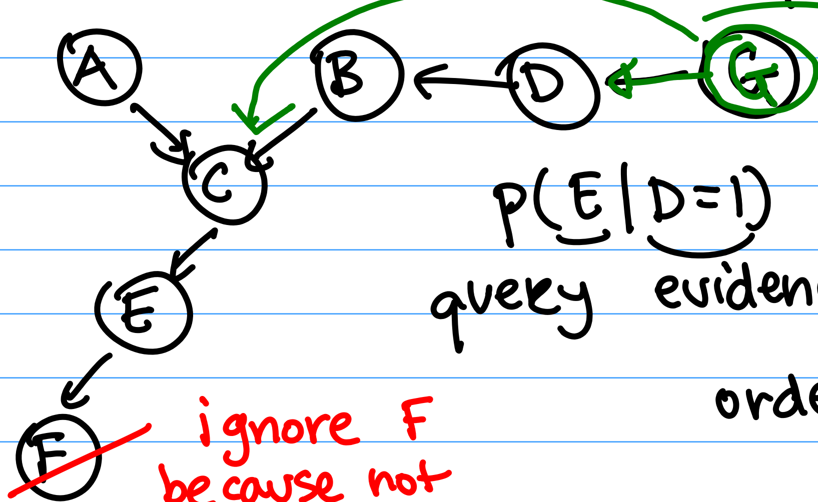

$$\underbrace{p(B}_{\text{query}}|\underbrace{A=1)}_{\text{evidence var}} \propto \sum_C p(B, A=1, C)$$

evidence var ↑ what you condition on

$$= \sum_C p(B|A=1)\,p(C|B)$$

$$= p(B|A=1) \underbrace{\sum_C \cancel{p(C|B)}}_{1}$$

② find leaves away from query (root) and work backwards toward query variable



if this edge existed, would be a lot harder!

$p(\underbrace{E}_{\text{query}} \mid \underbrace{D=1}_{\text{evidence}})$

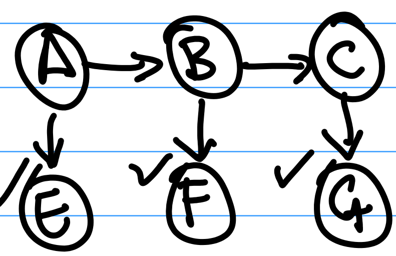
ignore F because not ancestor of E or D

ordering: A, B, C
B, A, C

$$p(E \mid D=1) = \sum_{A,B,C} p(A)\, p(B \mid D=1)\, p(C \mid A,B)\, p(E \mid C)$$

$$= \sum_{C} p(E \mid C) \sum_{A} p(A) \sum_{B} p(B \mid D=1)\, p(C \mid A,B)$$

## Another example:



$p(C \mid E,F,G)$

$$p(C \mid E,F,G) \propto \sum_{A,B} p(A)\, p(E{=}1 \mid A)\, p(B \mid A)\, p(F{=}1 \mid B)\, p(C \mid B)\, p(G{=}1 \mid C)$$

$$= \underbrace{p(G{=}1 \mid C)}_{K} \sum_{B} \underbrace{p(F{=}1 \mid B)}_{K}\, \underbrace{p(C \mid B)}_{K^2} \sum_{A} \underbrace{p(A)}_{K}\, \underbrace{p(B \mid A)}_{K^2}\, \underbrace{p(E{=}1 \mid A)}_{K}$$

size K $p(C \mid \cdot\cdot)$

pass on a size K $g(C)$

pass on a size K $g(B)$